Olimpiada de matematică – faza locală
Barem de corectare , clasa a XII-a

1.

a) Fie $u:(0,\infty)\to\mathbb{R}, u(x)=e^{2x}+3x^3+6x^2-12x+9$……..... 1p

$\int \frac{e^{2x}+x^3+5x^2-1}{e^{2x}+3x^3+6x^2-12x+9}dx=\frac{1}{3}\int \frac{u(x)+u'(x)}{u(x)}dx$…….……..…….. 2p

Finalizare…….....……..... 1p

b) $\int \frac{1}{x^4+x^{10}}dx=\int \frac{1}{x^{10}(\frac{1}{x^6}+1)}dx, u:(0,\infty)\to\mathbb{R}, u(x)=\frac{1}{x^3}$ ……1p

$\int \frac{1}{x^4+x^{10}}dx=-\frac{1}{3}\int \frac{u^2(x)u'(x)}{u^2(x)+1}dx=-\frac{1}{3}(u(x)-\int \frac{u'(x)}{u^2(x)+1}dx)$……...…….....……1p

Finalizare……...……1p

2.

a) Elementul neutru este $e=2$……...……..2p

Simetricul lui $a\in\mathbb{R}$ este $a'=f(2q-f^{-1}(a))$……..... 2p

b) $\sqrt[3]{x^2}+\sqrt[3]{x}=6$……....……2p

Finalizare……1p

3.

a) I_1=2……...1p

b) Şirul este monoton şi mărginit, deci convergent……...2p

$1\le\sqrt{1+sin^n x}\le 1+\frac{1}{2}sin^n x, \forall n\ge 1, \forall x\in[0,\frac{\pi}{2}]$ ……..2p

Rezultă $\frac{\pi}{2}\le I_n\le\frac{\pi}{2}+\frac{1}{2}J_n$, unde $J_n=\int_0^{\frac{\pi}{2}} sin^n x dx$ este şir care are limita egala cu zero.

Deci $\lim\limits_{n\to\infty} I_n=\frac{\pi}{2}$……..2p

4.

Fie $f:\mathbb{Z}_p\to\mathbb{Z}^*_p$ un morfism de grupuri.

Avem $f(\hat{0})=\hat{1}$ şi notăm $f(\hat{1})=\hat{a}$ 1p

$\hat{1}=f(\hat{0})=f(\hat{p})=f\left(\underbrace{\hat{1}+\cdots\hat{1}}_{p\ ori}\right)=f(\hat{1})f(\hat{1})\ldots f(\hat{1})=\hat{a}^p$.....2p

Conform teoremei lui Fermat rezultă că $\hat{a}^p=\hat{a}=\hat{1}$ deci$f(\hat{1})$=$\hat{1}$.....2p

Fie $\hat{x}\in\mathbb{Z}_p$. Atunci $f(\hat{x})=f\left(\underbrace{\hat{1}+\cdots\hat{1}}_{x\ ori}\right)=(f(\hat{1}))^x=\hat{1}$, deci unicul morfism este morfismul constant $f(\hat{x})=\hat{1}, \forall x\in\mathbb{Z}_p$.....2p